JPM UNILA
Jurnal Pendidikan Matematika Universitas Lampung
Vol. 14, No. 3, 2026, pp. 194 – 212
http://dx.doi.org/10.23960/mtk/v14i3.pp194-212
ISSN: 2715-856X

Designing a Hypothetical Learning Trajectory Based on Students' Learning Obstacles in the Transition from Arithmetic Thinking to Algebraic Thinking among Seventh-Grade Students

Mila Agustina[1], *Dedi Muhtadi[2], Sukirwan[3]

[1,2,3]Universitas Siliwangi, Jalan Siliwangi No.24, Tasikmalaya 46115, Indonesia
milaagustina321@gmail.com[1]; dedimuhtadi@unsil.ac.id[2*]; sukirwan@unsil.ac.id[3]

* corresponding author

ARTICLE INFO

Article history
Received: 13 July, 2026
Revised: 5 August, 2026
Accepted: 29 August, 2026
Published: 19 September, 2026

Keywords
algebraic thinking transition;
didactical design research;
hypothetical learning trajectory;
learning obstacles;
prospective analysis

ABSTRACT

This study aims to identify students' learning obstacles in the transition from arithmetic thinking to algebraic thinking and to design a Hypothetical Learning Trajectory based on these findings through a Didactical Design Research approach in the prospective analysis stage. This research employed an exploratory qualitative method involving 33 seventh-grade junior high school students selected through purposive sampling. Data were collected through diagnostic tests, teacher interviews, student interviews, and document analysis, and were analyzed using the Miles and Huberman model, consisting of data reduction, data display, and conclusion drawing. The findings reveal that students experience ontogenetic, epistemological, and didactical learning obstacles, particularly in understanding variables, coefficients, constants, algebraic term structures, and relationships among verbal, numerical, symbolic, and geometric representations. Based on these findings, a Hypothetical Learning Trajectory was developed, consisting of progressive learning stages from understanding the meaning of algebraic symbols, recognizing structural relationships among algebraic elements, connecting multiple representations, to performing substitution operations. In conclusion, the developed Hypothetical Learning Trajectory is anticipatory and grounded in students' learning obstacles, providing a systematic and adaptive foundation for mathematics instructional design that supports the development of students' algebraic thinking.

How to Cite: Agustina, M., Muhtadi, D., & Sukirwan. (2026). Designing a Hypothetical Learning Trajectory Based on Students' Learning Obstacles in the Transition from Arithmetic Thinking to Algebraic Thinking among Seventh-Grade Students. *Jurnal Pendidikan Matematika Universitas Lampung*, *14*(3), 194-212.
http://dx.doi.org/10.23960/mtk/v14i3.pp194-212

INTRODUCTION

Learning algebra at the junior high school level represents a crucial phase in students' mathematical thinking development, as it involves a transition from arithmetic thinking to more abstract and relational algebraic thinking (Blanton et al., 2015; Sun et al., 2023). This transition requires students' ability to understand symbols as representations of varying quantities and to connect multiple mathematical representations such as verbal, numerical, and symbolic forms within a meaningful

This is an open access article under the CC BY-SA 4.0 license.

structure (Sari et al., 2023; Stephens et al., 2017). However, various studies have shown that students still experience difficulties in understanding fundamental algebraic structures, particularly variables, coefficients, constants, and terms, indicating that the transition to algebraic thinking has not yet been fully achieved (Fauziah et al., 2023; Ruli et al., 2025). However, preliminary observations indicate that students have not yet understood the basic elements of algebra. Some students treat different variables as if they could be simplified with one another, indicating a limited understanding of variables as representations of unknown quantities. This condition indicates that algebra learning still faces challenges in developing students' relational understanding of basic algebraic concepts.

These difficulties can be understood as learning obstacles, which include epistemological, ontogenetic, and didactical obstacles. This study adopts Brousseau's theory of learning obstacles as the theoretical foundation for identifying and classifying these obstacles. According to Brousseau, learning obstacles may arise from limitations in students' prior knowledge and cognitive development (ontogenetic), the nature and contextualization of mathematical knowledge (epistemological), and the conditions of teaching and learning (didactical). Epistemological obstacles arise when students are unable to construct appropriate conceptual meanings of algebraic symbols, such as variables that are still perceived as fixed objects or mere letters (Duval, 2006; Uegatani et al., 2024). Ontogenetic obstacles are related to students' cognitive readiness, which is still dominated by arithmetic thinking, making it difficult for them to generalize (Anggraeni et al., 2024; Habsy et al., 2023). Meanwhile, didactical obstacles emerge from instructional practices that remain procedural and provide limited exploratory experiences in understanding the meaning of algebraic structures (Fasya et al., 2024; Santosa et al., 2024). These three obstacles indicate that difficulties in learning algebra result from a complex interaction among students, subject matter, and instructional design, thus requiring a more systematic learning design approach.

Within the framework of Didactical Design Research (DDR), particularly in the prospective analysis stage, this approach was chosen because the study aims not only to identify students' learning obstacles but also to use these obstacles as a basis for developing an instructional design that anticipates students' learning processes. The research process not only focuses on identifying learning difficulties but also produces two interrelated main products, namely students' actual learning obstacles (LO) and the Hypothetical Learning Trajectory (HLT) (Apriyanti et al., 2019). Actual LO functions as an empirical representation of the gap between formal algebraic concepts and students' understanding obtained through diagnostic tests, interviews, and learning analysis (Fasya et al., 2024). HLT is subsequently developed as a hypothetical learning trajectory that

uses actual LO as an epistemic foundation to anticipate and respond to students' learning needs through systematic learning stages (Apriyanti et al., 2019; Faza et al., 2025; Hendrik et al., 2020). Thus, HLT is not merely a sequence of instructional activities but a pedagogical construction grounded in the analysis of learning obstacles and learning needs (Bakker, 2018; Tondorf & Prediger, 2022).

Several previous studies have developed HLT in various mathematics learning contexts, such as the algebraic multiplication using the Realistic Mathematics Education (RME) approach and using the Conceptual Understanding Procedures (CUPs) model (Pramesti et al., 2025; Yuliani et al., 2024). However, the reviewed studies do not explicitly position empirically identified actual learning obstacles as the primary basis for constructing the HLT. In particular, the systematic process of connecting theoretical learning obstacles, actual learning obstacles, learning needs, and HLT within the DDR framework at the prospective analysis stage has not been explicitly addressed (Rosita et al., 2024; Sukarma et al., 2024). Without grounding HLT in students' actual learning obstacles, the resulting learning trajectory may be less responsive to their specific difficulties and learning needs. Therefore, this study addresses this gap by developing an HLT grounded in empirically identified learning obstacles in the transition from arithmetic thinking to algebraic thinking.

Based on this, the present study aims to identify students' learning obstacles in the transition to algebraic thinking and to develop a Hypothetical Learning Trajectory (HLT) based on the results of this analysis within the DDR framework at the prospective analysis stage (Apriyanti et al., 2019). This study emphasizes that actual LO and HLT are two interrelated main products in didactical design processes, thereby contributing to the development of more adaptive, systematic, and responsive mathematics learning that supports students' algebraic thinking development (Sun et al., 2023).

METHOD

This study employed Didactical Design Research (DDR), focusing on the prospective analysis stage. The research was conducted at SMPN 12 Tasikmalaya, involving 33 seventh-grade students from class VII-F who had studied algebraic expressions. The participants were selected through purposive sampling based on their prior experience with algebraic expressions and preliminary indications of learning obstacles identified from the mathematics teacher's information and students' previous learning outcomes. Class VII-F was selected because the mathematics teacher identified the class as having heterogeneous cognitive characteristics, including students with varying levels of mathematical ability and a mathematics teacher as a supporting informant to provide contextual information about students' previous learning

experiences, difficulties in learning algebraic expressions, and instructional practices. Data were collected through a diagnostic test, semi-structured student interviews, teacher interviews, and documentation. The diagnostic test consisted of four open-ended questions, with one question containing two sub-questions, covering indicators related to algebraic elements, variables, coefficients, constants, terms, relationships among algebraic elements, verbal, geometric, and symbolic representations, and substitution. The test was developed by the researchers in consultation with two mathematics education lecturers based on the theoretical learning obstacles and the textbook used by the students, and was validated by two other mathematics education lecturers through expert validation, followed by one revision.

After the diagnostic responses were analyzed, five students whose responses most clearly indicated learning obstacles were selected for semi-structured interviews. The interviews explored students' understanding, reasoning, and the basis of their responses to the diagnostic tasks. Documentation included students' answer sheets, teaching materials, textbooks, teacher notes, and photographs of the research activities. The diagnostic test consisted of four open-ended questions, with one question containing two sub-questions, covering the theoretical learning obstacle indicators. It was developed by the researchers in consultation with two mathematics education lecturers based on the theoretical learning obstacles and the students' textbook, and validated by two other lecturers through expert validation followed by one revision. After the diagnostic responses were analyzed, five students whose responses most clearly indicated learning obstacles were selected for the semi-structured interviews to explore their understanding and reasoning. All data were analyzed using the Miles and Huberman model, comprising data reduction, data display, and conclusion drawing (Miles et al., 2014). Learning obstacles were identified through conceptual analysis, theoretical LO, diagnostic responses, and interview data, and classified as epistemological, ontogenetic, or didactical obstacles based on Brousseau's theory, considering students' prerequisite knowledge, the nature of algebraic concepts, and instructional conditions. The identified actual LO and learning needs were then used to develop the HLT.

RESULTS AND DISCUSSION

Algebraic expressions at the junior secondary school level constitute an important foundation for the development of algebraic thinking, emphasizing the transition from arithmetic reasoning to symbolic representation. In this study, the conceptual structure of algebra includes variables, coefficients, constants, and terms, as well as the relationships among these elements in forming meaningful algebraic expressions. In addition, algebra is viewed as a representational system that connects verbal, numerical, and symbolic

forms through substitution processes and algebraic notation rules. The complexity of these interrelated concepts makes algebra an important context for identifying students' learning obstacles.

Prior to the empirical analysis, theoretical learning obstacles were formulated through a synthesis of conceptual analysis and literature review on common student difficulties in algebraic expressions. The synthesis connected the structure of algebraic concepts with learning difficulties reported in previous studies to identify potential learning obstacles and classify them according to Brousseau's learning obstacle framework.

Table 1. Theoretical Learning Obstacles Analysis

Code	Concept	Predicted Learning Obstacle	Type
C1.1	Algebraic elements	Students memorize without understanding algebraic elements	Epistemological
C1.2	Algebraic elements	Variables are interpreted as fixed objects or labels	Epistemological
C2	Relations among elements	Operations are performed without understanding structural relationships	Epistemological
C3.1	Verbal representation	Incorrect linking between verbal context and algebraic structure	Epistemological
C3.2	Geometric representation	Area is interpreted as the sum of side lengths	Epistemological
C3.3	Geometry–algebra representation	Inability to connect different representations	Didactical
C4	Symbolic representation	Difficulty in applying algebraic notation rules	Didactical
C5	Substitution	Difficulty in determining values of algebraic expressions	Didactical

Based on the diagnostic test conducted on students' learning difficulties in algebra, the findings are described as follows.

Concept 1: Algebraic Elements

C1.1. Identifying Algebraic Elements

Question: Determining variables from real-life context.

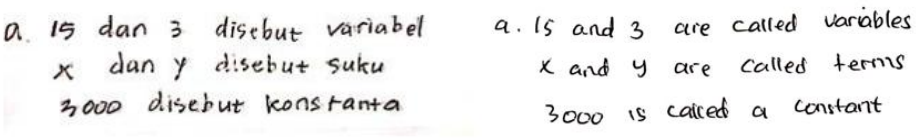


Figure 1. Students' difficulties in C1.1

A total of 27.2% of students were unable to correctly identify variables, coefficients, constants, and terms. Students tended to misinterpret algebraic elements, such as treating numbers as variables, indicating weak understanding of symbolic

structure. Interview results show that students mainly memorize algebraic forms without understanding the function of each element, while the teacher stated that instruction is still procedurally oriented. This condition reflects dominance of arithmetic-based thinking.

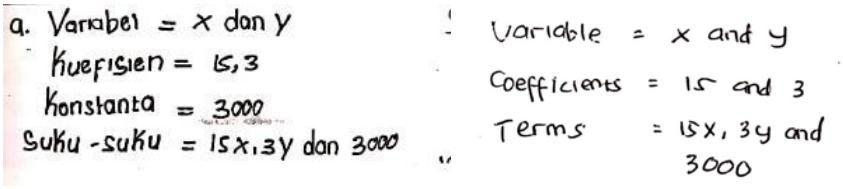


Figure 2. Students' difficulties in C1.1 (LON 1)

Learning Obstacle New 1 (LON 1) at 24.2% shows that students do not understand positive and negative signs as part of term values, but only as operational symbols. This indicates weak symbolic meaning in algebraic structure.

C1.2 Understanding Variables
Question: Determining variables from real-life context.
The price of one pencil is x rupiah. Dita buys 5 pencils and gets a discount of Rp1,000.

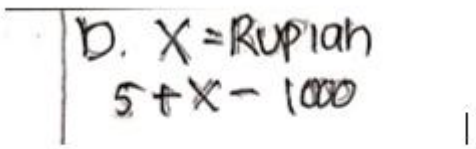
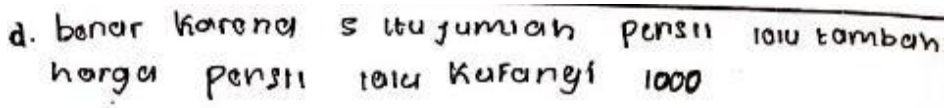


Figure 3. Students' difficulties in C1.2

A total of 24.2% of students did not understand variables as representations of changing quantities. Variables are still interpreted as fixed values that must be known. This indicates that variables have not been understood as relational entities in algebra, reflecting arithmetic-based thinking dominance

Concept 2: Relations Among Algebraic Elements
Question: Determine the algebraic expression from a real situation. "The total price to be paid is 5 + x − 1000."



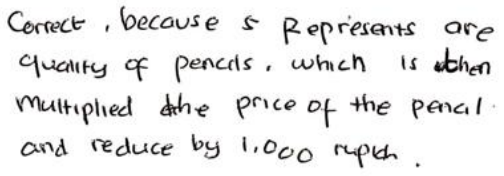


Figure 4. Students' difficulties in C2

A total of 81.8% of students were unable to understand structural relationships among algebraic elements. Students processed each element separately without understanding the overall structure.

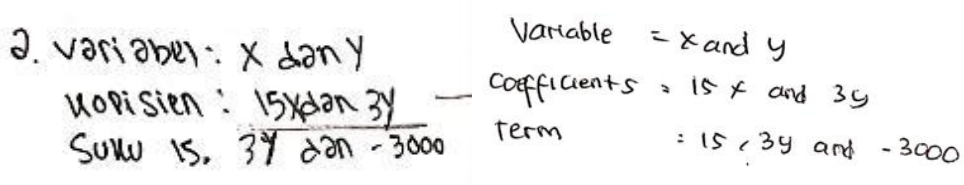


Figure 5. Students' difficulties in C2 (LON 2)

Learning Obstacle 2 (LON 2) at 12% shows that students do not distinguish coefficients and variables structurally. This indicates that internal structure of algebraic terms has not been formed. Interview results show that students perceive "15x" as a single unit, while teachers stated that students struggle with basic algebraic structure. This reflects lack of structural decomposition.

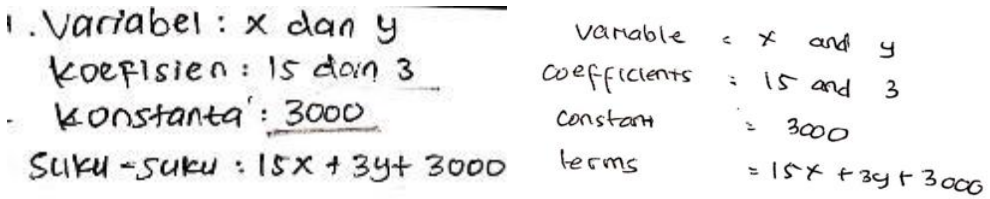


Figure 6. Students' difficulties in C2 (LON 3)

Learning Obstacle 3 (LON 3) at 6% shows that students perceive expressions as undivided units without term segmentation. Interviews indicate that students only copy expressions without structural breakdown, while teachers report weak understanding of expression structure. This reflects lack of symbolic segmentation ability.

Concept 3: Algebra as Representation of Quantity Relations
C3.1 Contextual Representation
Question: Create a real-life situation from $15x + 3y - 3000$.

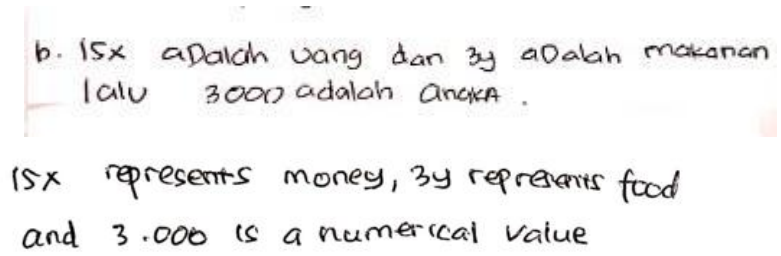


Figure 7. Students' difficulties in C3.1

A total of 63.6% of students were unable to translate algebraic expressions into meaningful real-life contexts. Students tended to create irrelevant stories that do not represent quantitative relationships. Interviews show that students struggle to connect symbols with context, while teachers stated that contextual problems are rarely explored in depth. This indicates weak representational fluency.

C3.2 Geometric Representation

Question: Determine the area of a rectangle with length (3x + 5)cm and width (2x + 8) cm.

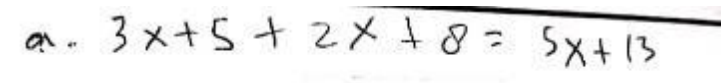


Figure 8. Students' difficulties in C3.2

A total of 21.2% of students were unable to relate length and width in algebraic form to determine area correctly. Students often used incorrect operations such as addition. The concept of multiplication as area representation has not been internalized. Interviews show students lack conceptual understanding of area, while teachers noted weak prerequisite knowledge. This reflects lack of coordination between algebraic and geometric representation.

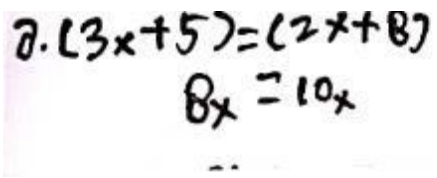


Figure 9. Students' difficulties in C3.2 (LON 4)

Learning Obstacle 4 (LON 4) at 18% shows incorrect interpretation of length, width, and area relationships, leading to failure in geometric–algebraic mapping. Interviews indicate students do not understand area relationships, while teachers report frequent errors in geometry modeling. This reflects weak structural mapping between representations.

C3.3 Representation Flexibility
Question: Alternative representation of area using geometric approach.

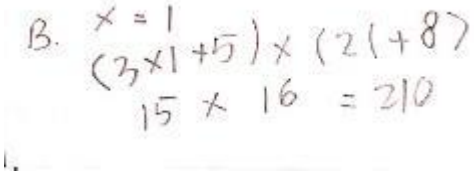


Figure 10. Students' difficulties in C3.3

A total of 84.8% of students failed to recognize that a concept can be represented in equivalent forms. Interviews show that students rely only on textbook examples, while teachers stated limited exploration of alternative representations. This indicates low representational flexibility in algebra.

Concept 4: Symbolic Representation
Question: Rewrite (h × (h × 5 × r) − 8) × (h) using algebraic rules.

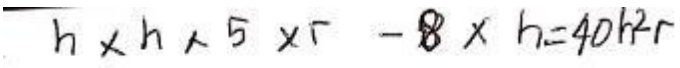


Figure 11. Students' difficulties in C4

A total of 75.7% of students experienced difficulties applying algebraic notation rules, especially in nested expressions. Interviews show that students can perform initial substitution but struggle in further operations, while teachers stated that the main weakness lies in arithmetic operations. This reflects lack of integration between procedural and structural reasoning.

Concept 5: Value of Algebraic Expressions
Question: Determine the value of (h × (h × 5 × r) − 8) × (h), if h = 2 and r = 1.

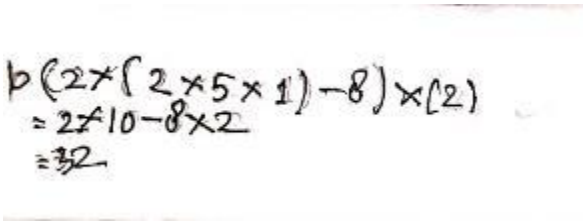


Figure 12. Students' difficulties in C5

A total of 51.5% of students made errors in substituting variable values, especially in operations after substitution. Interviews show that students can substitute values but struggle in further calculations, while teachers stated that errors mainly occur in basic operations. This indicates that substitution is not yet understood as a structured algebraic process. Based on students' responses and empirical findings confirmed through interviews with teachers and students, the actual learning obstacle table is presented in the next section.

Table 2. Actual Learning Obstacles

Concept	Sub Concept	%	Actual Learning Obstacle	Type
C1.Algebraic elements	C1.1	27.2%	Difficulty identifying algebraic elements	Epistemological & Didactical
		24.2%	Weak understanding of positive and negative signs in algebraic terms	Epistemological & Didactical
C2.Relations among elements	C1.2	24.2%	Variables not understood as dynamic quantities	Epistemological & Didactical
	C2	81.8%	Lack of relational understanding among algebraic elements	Epistemological & Didactical
		12%	Inability to decompose terms into coefficients and variables	Epistemological & Didactical
		6%	Expressions perceived as undivided global structures	Epistemological & Didactical
C3.Algebraic representation	C3.1	63.6%	Failure to translate algebra into contextual representation	Didactical
	C3.2	21.2%	Difficulty representing length–width–area relationships	Epistemological & Didactical
		18%	Area not understood as multiplication of two quantities	Ontogenetic & Epistemological
	C3.3	84.8%	Difficulty connecting algebraic and geometric representations	Ontogenetic, Epistemological & Didactical
C4.Symbolic representation	C4	75.7%	Difficulty applying algebraic notation rules consistently	Didactical
C5.Algebraic value	C5	51.5%	Difficulty integrating substitution with algebraic structure	Ontogenetic & Epistemological

The identified actual learning obstacles indicate students' difficulties in understanding algebraic concepts and the characteristics of their errors; however, they do not directly specify the instructional design required to address them. Therefore, a learning needs analysis is necessary to identify the knowledge, experiences, and cognitive processes that need to be facilitated in learning. In this study, the analysis is conducted through a synthesis of conceptual analysis and actual learning obstacles obtained from diagnostic tests and interviews. Thus, learning needs are formulated as learning conditions that support the achievement of conceptual understanding while simultaneously addressing the identified obstacles.

Table 3. Learning Needs Analysis

Code	Actual Learning Obstacle	Interpretation of the Obstacle	Learning Needs
C1.1	Difficulty understanding and distinguishing algebraic elements	Learning is based on memorization	Identifying algebraic elements based on their meaning
C1.2	Difficulty understanding variables as dynamic quantities	Variables are seen as fixed symbols	Understanding variables as changing quantities
C2	Difficulty understanding relations among algebraic elements	Relationships among elements are not understood	Analyzing relationships among algebraic elements
C3.1	Difficulty representing algebra in context	Quantitative relationships are not understood	Connecting context with algebraic expressions
Code	**Actual Learning Obstacle**	**Interpretation of the Obstacle**	**Learning Needs**
C3.2	Difficulty representing length, width, and area relationships	Multiplicative relationship is not understood	Linking rectangle area concept with algebraic representation
C3.3	Difficulty understanding equivalence of representations	Representations are not understood as equivalent forms	Exploring equivalent algebraic and geometric representations
C4	Difficulty applying rules of algebraic notation consistently	Rules are not fully understood	Applying algebraic notation meaningfully
C5	Difficulty performing substitution and subsequent operations	Operations after substitution are not structured	Performing substitution and operations according to algebraic structure
LON1	Difficulty interpreting positive and negative signs	Signs are not understood as part of value	Understanding the function of signs in algebra
LON2	Difficulty distinguishing coefficients and variables	Elements are not structurally separated	Distinguishing and explaining coefficients and variables

LON3	Difficulty understanding terms in algebraic expressions	Expressions are seen as a single unit	Identifying and understanding algebraic terms
LON4	Difficulty understanding area as multiplication of length and width	Geometric–algebraic relationship is not formed	Understanding area as multiplication in algebra

Based on the learning needs analysis, the next stage is to develop a Hypothetical Learning Trajectory (HLT) as a hypothetical learning sequence that illustrates the progression of students' understanding in learning algebraic expressions. It is constructed based on the interrelationships among concepts within the material structure as well as the learning needs identified from the actual learning obstacles. Thus, each stage of the HLT is not designed to address a single obstacle separately, but rather represents an integration of various learning needs that are conceptually interconnected and gradually facilitated in accordance with Kieran's theory.

Tabel 4. Hypothetical Learning Trajectory (HLT)

HLT Stage	Learning Development Objective	Learning Needs	Predicted Student Responses	ADP
HLT-1 (C1.2)	Understanding variables as dynamic quantities	Constructing variables as changing quantities depending on context, not fixed values	Students view symbols as fixed labels that do not change across contexts.	The teacher directs students to compare values in a table to observe data inconsistency over time. Question: *"What happens to book prices from week to week?"* The teacher emphasizes that symbols represent quantities that change depending on the context
HLT-2 (LON2)	Distinguishing coefficients and variables in algebraic structure	Identifying term structure as coefficient (multiplier) and variable	Students see algebraic expressions as single units and cannot distinguish coefficients and variables.	The teacher guides students to decompose algebraic expressions step by step. Question: *"Which part represents the number of groups and which part represents the content of each group?"* The teacher emphasizes that one part represents the multiplier and the other represents the repeated quantity.
HLT-3 (LON1)	Understanding the meaning of positive and negative signs in terms	Treating signs as part of term value, not only as operations	Students ignore signs and interpret positive and negative values as identical.	The teacher creates cognitive conflict by comparing values with different signs. Question: *"What do positive and negative signs mean in bank balances?"* The teacher emphasizes that signs are part of value, not merely operational symbols.
HLT-4 (LON3,C1.1)	Identifying algebraic terms and elements	Segmenting expressions into terms and understanding algebraic elements	Students fail to identify term boundaries and confuse	The teacher guides students to identify structural parts of expressions based on operation signs. Question: *"Is this*

HLT Stage	Learning Development Objective	Learning Needs	Predicted Student Responses	ADP
			variables and coefficients.	*expression made up of one part or several parts?"* The teacher emphasizes that each part separated by operations is called a term.
HLT-5 (C2)	Understanding relationships among algebraic elements	Understanding relationships between coefficients, variables, and operations	Students use addition for all operations and do not understand repeated multiplication.	The teacher guides students to convert repeated addition into a simplified algebraic form. Question: *"What is the same and what changes?"* The teacher emphasizes that coefficients represent repeated multiplication of variables.
HLT-6 (C3.1)	Translating context into algebraic expressions	Maintaining relational structure when translating context to symbols	Students construct models without maintaining relationships among quantities.	The teacher directs students to identify relevant information before modeling. Question: *"What information is given in the situation?"* The teacher emphasizes that all elements must remain connected within the algebraic model.
HLT-7 (LON4)	Understanding area as multiplication of length and width	Representing geometric relationships consistently in algebraic form	Students incorrectly equate different quantities (e.g., length = width) instead of using algebraic relations.	The teacher guides analysis of the relationship between length and width. Question: *"How are length and width related in a rectangle?"* The teacher emphasizes that area represents a relationship, not an addition of quantities.
HLT-8 (C3.2)	Representing geometric shapes algebraically	Constructing algebraic models from geometric relationships	Students use addition instead of multiplication for area concepts.	The teacher connects unit-square models with symbolic representation. Question: *"Is the relationship addition or*

HLT Stage	Learning Development Objective	Learning Needs	Predicted Student Responses	ADP
				multiplication?" The teacher emphasizes that area is represented through multiplication.
HLT-9 (C3.3)	Understanding equivalence of representations	Recognizing that one concept can have equivalent representations	Students assume only one correct representation and reject alternatives	The teacher facilitates verification of representation equivalence. Question: *"Is there another way to express this area?"* The teacher emphasizes that different representations are equivalent if the quantitative relationship remains the same.
HLT-10 (C4)	Understanding algebraic notation rules	Using algebraic notation with correct structure and hierarchy	Students memorize forms without understanding notation rules or meanings.	The teacher discusses consistency in algebraic notation. Question: *"Do all these forms have the same meaning?"* The teacher emphasizes that notation rules are mathematical conventions to avoid ambiguity.
HLT-11 (C5)	Understanding substitution in algebraic expressions	Performing substitution without changing expression structure	Students substitute values but fail to maintain expression structure.	The teacher guides substitution while preserving structural integrity. Question: *"What happens when a variable is replaced by a value?"* The teacher emphasizes that substitution does not change the structural relationship within the expression.

Overall, the developed HLT demonstrates a progressive learning trajectory from basic understanding toward structural and relational understanding of algebraic expressions. This progression indicates that students' transition from arithmetic to algebraic thinking requires more than learning symbolic procedures; it requires a gradual reconstruction of the meanings and relationships underlying algebraic representations. The initial stage (HLT-1 to HLT-3) focuses on strengthening the basic meaning of algebraic elements, namely variables as dynamic quantities, the distinction between

coefficients and variables, and the interpretation of positive–negative signs as part of value, which addresses C1.2, LON 2, and LON 1. These stages respond to the identified tendency of students to interpret algebraic symbols as fixed or undifferentiated objects, suggesting that meaning-making should precede formal manipulation. The subsequent stage (HLT-4 to HLT-11) emphasizes the development of structural and relational understanding, including term segmentation (LON 3 and C1.1), coefficient–variable relationships (C2), translation across representations (C3.1), and the connection between equivalent geometric and algebraic representations (LON 4, C3.2, C3.3). This sequence reflects the interdependence among algebraic concepts and representations, in which understanding the structure of expressions provides a foundation for establishing relationships across symbolic, verbal, and geometric forms. The final stage highlights formalization through symbolic rules and substitution (C4 and C5). Thus, the HLT extends beyond a content sequence by translating students' actual learning obstacles into interconnected learning needs and progressively organized instructional stages. This provides a distinctive contribution of the present study: the HLT is constructed from empirically identified learning obstacles in the arithmetic-to-algebra transition rather than being organized solely around a predetermined instructional approach. Therefore, the HLT does not merely sequence learning content, but reconstructs students' thinking from arithmetic reasoning toward more relational and meaningful algebraic understanding.

CONCLUSION

Based on the research findings, it can be concluded that students experienced learning obstacles in algebraic expressions, indicated by difficulties in understanding symbolic meaning, relationships among algebraic elements, and connections between multiple mathematical representations. These obstacles are reflected in students' understanding of variables, coefficients, constants, and terms, interpretation of positive and negative signs, difficulties in identifying expression structures, as well as errors in linking algebraic representations with verbal and geometric contexts and in applying notation rules and substitution. These findings indicate that students are still in the transition from arithmetic thinking toward more formal and relational algebraic thinking. Based on these actual learning obstacles, learning needs were identified and subsequently translated into a Hypothetical Learning Trajectory (HLT) through systematic instructional design. This LO–learning needs–HLT sequence addresses the identified research gap by positioning empirically identified learning obstacles as the foundation for constructing the HLT, rather than treating the learning trajectory solely as a predetermined sequence of content or instructional activities. The HLT is structured from understanding algebraic symbols and elements, relationships among elements,

connections between representations (verbal, symbolic, and geometric), to the formalization of notation rules and substitution. Thus, the HLT is designed to facilitate students' thinking development from informal understanding toward formal, relational, and meaningful algebraic understanding.

Practically, teachers are recommended to integrate contextual learning, multiple representations, and conceptual understanding in algebra instruction. This study is limited to the prospective analysis stage; therefore, the developed HLT has not yet been implemented and empirically evaluated in classroom learning. Future research should implement and evaluate the HLT to examine its feasibility, students' learning progression, and its effectiveness in addressing the identified learning obstacles, as well as explore additional learning obstacles to further enrich the didactical design of algebra learning.

REFERENCE

Anggraeni, N. D., Kumala Dwi Alviana, W., Fitriya Wahyuni, D., Kusuma Ainurrosyidah, L. D., Mahardika, I. ketut, Sutarto, S., & Wicaksono, I. (2024). Analisis Perkembangan Peserta Didik Menurut Teori Jean Piaget dan Pengimplementasian pada Pembelajaran IPA SMP. *Edusaintek: Jurnal Pendidikan, Sains dan Teknologi*, 11(3), 1503–1519. https://doi.org/10.47668/edusaintek.v11i3.1252

Apriyanti, M., Suweken, G., & Suparta, I. N. (2019). Development of hypothetical Learning Trajectory for Linear Equations with PISA and Scientific Approach Model Consideration. *Jurnal Pengajaran Matematika dan Ilmu Pengetahuan Alam*, 24(2), 35–43. https://doi.org/10.18269/jpmipa.v24i2.15856

Bakker, A. (2018). *Design Research in Education*. London: Routledge. https://doi.org/10.4324/9780203701010

Blanton, M., Stephens, A., Knuth, E., Gardiner, A. M., Isler, I., & Kim, J.-S. (2015). The Development of Children's Algebraic Thinking: The Impact of a Comprehensive Early Algebra Intervention in Third Ggrade. *Journal for Research in Mathematics Education*, 46(1), 39–87. https://doi.org/10.5951/jresematheduc.46.1.0039

Duval, R. (2006). A Cognitive Analysis of Problems of Comprehension in a Learning of Mathematics. *Educational Studies in Mathematics*, 61(1–2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Fasya, R. I., Suryadi, D., & Nurjanah, N. (2024). Learning Obstacle Siswa Pada Materi Operasi Hitung Bentuk Aljabar Melalui Soal Pemecahan Masalah Matematika. *Prismatika: Jurnal Pendidikan dan Riset Matematika*, 7(1), 112–123. https://doi.org/10.33503/prismatika.v7i1.198

Fauziah, E., Lidinillah, D. A. M., & Apriani, I. F. (2023). Obstacle to Learning Algebra in Elementary Schools. *AlphaMath: Journal of Mathematics Education*, 9(2), 161. https://doi.org/10.30595/alphamath.v9i2.18612

Faza, N., Eliza, R., & Khaidir, C. (2025). The Development of Hypothetical Learning Trajectory (HLT) Based on Problem-Based Learning (PBL) in The Concept of Relations and Functions. *AXIOM: Jurnal Pendidikan dan Matematika*, 14(1), 87. https://doi.org/10.30821/axiom.v14i1.19699

Habsy, B. A., Christian, J. S., M, S. U. S. P., & Unaisah, U. (2023). Memahami Teori Pembelajaran Kognitif dan Konstruktivisme serta Penerapannya. *TSAQOFAH*, 4(1), 308–325. https://doi.org/10.58578/tsaqofah.v4i1.2177

Hendrik, A. I., Ekowati, C. K., & Samo, D. D. (2020). Kajian Hypothetical Learning Trajectories dalam Pembelajaran Matematika di Tingkat SMP. *Fraktal: Jurnal Matematika dan Pendidikan Matematika*, 1(1), 1–11. https://doi.org/10.35508/fractal.v1i1.2683

Miles, M. B., Huberman, A. M., & J, S. (2014). *Qualitative Data Analysis (3rd ed., Vol. 17)*. United States: Sage.

Pramesti, G., Nastiti, B. T. Y., & Wiraya, A. (2025). Developing a Learning Trajectory on the Conceptual Understanding Procedures of Pythagorean Theorem Material for Junior High School Students. *KnE Social Sciences*, 10(11), 202–219. https://doi.org/10.18502/kss.v10i11.18742

Rosita, C. D., Maulina, Q. T., Sa'diyyah, H., Hartono, W. (2024). Systematic Literature Review Related to Learning Obstacles of Junior High School Students in Understanding Algebra Concepts. *International Journal of Humanities Education and Social Sciences (IJHESS)*, 4(1). 518-553. https://doi.org/10.55227/ijhess.v4i1.1121

Ruli, R. M., Juandi, D., Dahlan, J. A., & Maudy, S. Y. (2025). From Arithmetic to Algebra: Students' Epistemological Obstacles. *SJME (Supremum Journal of Mathematics Education)*, 9(2), 371–382. https://doi.org/10.35706/sjme.v9i2.205

Santosa, F. H., Indrawati, I., & Bahri, S. (2024). Hambatan belajar siswa pada materi kaidah pencacahan. *Journal of Math Tadris*, 4(2), 78–96. https://doi.org/10.55099/jmt.v4i2.158

Sari, I. K., Nasriadi, A., Amalia, Y., Musfiya, M., & Setiawati, F. A. (2023). Middle School Students' Mathematical Representational Ability: A Case Study of the Development of Students' Cognitive Ability through the Realistic Mathematics Education Approach. *Mosharafa: Jurnal Pendidikan Matematika*, 12(3), 453–464. https://doi.org/10.31980/mosharafa.v12i3.2556

Stephens, A. C., Ellis, A. B., Blanton, M., & Brizuela, B. M. (2017). Algebraic Thinking in the Elementary and Middle Grades. *Compendium for Research in Mathematics Education*, 386–420.

Sukarma, I. K., Isnawan, M. G., & Alsulami, N. M. (2024). Research on Nonroutine Problems: A Hybrid Didactical Design for Overcoming Student Learning Obstacles. *Human Behavior and Emerging Technologies*, 2024, 1–12. https://doi.org/10.1155/2024/5552365

Sun, S., Sun, D., & Xu, T. (2023). The Developmental Progression of Early Algebraic Thinking of Elementary School Students. *Journal of Intelligence*, 11(12), 1-19. https://doi.org/10.3390/jintelligence11120222

Tondorf, A., & Prediger, S. (2022). Connecting Characterizations of Equivalence of Expressions: Design Research in Grade 5 by Bridging Graphical and Symbolic Representations. *Educational Studies in Mathematics*, 111(3), 399–422. https://doi.org/10.1007/s10649-022-10158-0

Uegatani, Y., Otani, H., Shirakawa, S., & Ito, R. (2024). Real and Illusionary Difficulties in Conceptual Learning in Mathematics: Comparison Between Constructivist and Inferentialist Perspectives. *Mathematics Education Research Journal*, 36(4), 895–915. https://doi.org/10.1007/s13394-023-00478-6

Yuliani, S. B., Palupi, E. L. W., & Fiangga, S. (2024). Pengembangan Hypothetical Learning Trajectory Materi Perkalian Bentuk Aljabar Kelas VII dengan Pendekatan Realistic Mathematics Education. *Mathedunesa*, 13(2), 550–575. https://doi.org/10.26740/mathedunesa.v13n2.p550-575